

An SMT-based Approach for Generating Coverage Oriented Metamodel Instances

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Abstract: An effective technique for generating instances of a metamodel should quickly and automatically generate instances satisfying the metamodel's structural and OCL constraints. Ideally it should also produce quantitatively meaningful instances with respect to certain criteria, that is, instances which meet specified generic coverage criteria that help the modelers test or verify a metamodel at a general level. In this paper, we present an approach consisting of two techniques for coverage oriented metamodel instance generation. The first technique realises the standard coverage criteria defined for UML class diagrams, while the second technique focuses on generating instances satisfying graph-based criteria. With our approach, both kinds of criteria are translated to SMT formulas which are then investigated by an SMT solver. Each successful assignment is then interpreted as a metamodel instance that provably satisfies a coverage criteria or a graph property. We have already integrated this approach into our existing tool to demonstrate the feasibility.

1 Introduction

A *model* provides a representation of aspects of a system. This can include design models such as UML class or sequence diagrams, or implementation models, such as source code in a programming language. A *metamodel* is a model that is used to describe the structure of other models, modelling languages or domain specific languages. Each *instance* of a metamodel is then a model that can be regarded as a test case. These test cases are important not just for validating a metamodel itself, but also useful for testing the tools and frameworks that process the models defined by that metamodel such as model transformation.

For example, given a domain specific language L , say, a metamodel would usually define the abstract syntax and static semantics of the language. A typical representation of the metamodel would be as a UML class diagram (using a subset of the constructs) with constraints specified using the Object Constraint Language (OCL). A set of instances of this metamodel would be programs written in language L , and would allow language engineers to check that they had specified the relevant constructs correctly.

A number of approaches and tools have already

provided a way of generating these instances (Ehrig et al., 2009; González Pérez et al., 2012; Cabot et al., 2014). However, these instances are not measured via any criteria. At least, meeting some criteria such as standard coverage criteria for UML class diagram would help users to increase their confidence in designing or validating metamodels. Furthermore, users may also wish to generate instances that possess certain coverage metrics for other testing purposes such as using depth of inheritance tree for testing inheritance relationships. Thus, naively generating instances from a metamodel without taking account of coverage criteria or other properties is not very adequate.

This paper addresses the issue of generating metamodel instances satisfying coverage criteria. More specifically, this paper makes the following contributions:

- A technique that enables metamodel instances to be generated so that they satisfy partition-based coverage criteria.
- A technique for generating metamodel instances which satisfy graph properties.

Both two techniques that encode coverage criteria and graph properties into a set of SMT formulas. These

formulas are then combined with the formulas generated from our previous work, and solved by using an external SMT solver (Wu et al., 2013). Each successful assignment for the formulas is interpreted as an instance. We have already automated this process into a tool to demonstrate the feasibility of this approach.

2 Background

In this section, we briefly review the standard coverage criteria defined for UML class diagram, notations we use for expressing a metamodel as a graph, and basic SMT encodings from our previous work. Formally, we consider all metamodels in this paper as being presented as UML class diagrams, and represented as graphs.

2.1 Metamodel Coverage Criteria

A metamodel is a structural diagram and can be depicted using the UML class diagram notation. Thus, the coverage criteria defined for UML class diagram can also be borrowed for metamodels. In particular, we focus on the coverage criteria presented in (Andrews et al., 2003) (Ghosh et al., 2003), especially the work focused on testing the structural elements of a UML class diagram. These coverage criteria are standard criteria for testing a UML class diagram and they are defined as follows:

- Generalisation coverage (*GN*) which describes how to measure inheritance relationships.
- Association-end multiplicity coverage (*AEM*) which measures association relationships defined between classes.
- Class attribute coverage (*CA*) which measures the set of representative attribute value combinations in each instance of class.

AEM and *CA* are partition-based testing criteria which means that testing results depend on the choice of a representative value from each partition (Ostrand and Balcer, 1988). Therefore, the value domain is partitioned into several equivalence classes, and each value from an equivalent class is expected to have the same results. The partitions can also be decided using domain knowledge-based partitioning.

For example, to satisfy the *CA* criterion for the metamodel in Figure 1, we may assume a user could choose a representative value of 18¹, and this allows

¹A user may choose a different representative value, this depends on the knowledge about a specific domain.

the attribute *age* in the abstract class *Person* to be divided into 3 partitions which are $18 < age$, $age = 18$ and $age > 18$. The hypothesis is that any single value from one of the three partitions is expected to have the same results for all other values from that partition (Myers and Sandler, 2004). Similarly, to satisfy the *AEM* criterion, the binary association (*employees*) in the metamodel can also be divided into two partitions: a *Department* that has no *Manager* or a *Department* that has multiple *Managers*. The multiple number of *Managers* can be a boundary value chosen by a user. For example, it can be the maximum value that an integer can hold or 5 if it is determined by specific domain knowledge about the model. Finally, to satisfy the *GN* criterion, the inheritance relation can be covered by ensuring the creation of an instance of *Manager*.

In this paper, we focus on generating instances meeting *CA* and *AEM* criteria by providing a general SMT encoding. For *GN*, it has already been incorporated into our previous work. Our previous work takes a metamodel, presented as a class diagram with OCL constraints, augmented with quantitative constraints, and uses an SMT solver to generate instances. Our earlier work also supports a subset of OCL, this includes: constraints on an attribute, navigation over an association, and nested quantifiers over a collection of instances (Wu et al., 2013). To facilitate the transformation from class diagrams with OCL constraints to SMT formulas we use a *bounded typed graph* as an intermediate representation.

2.2 Bounded Typed Graphs

Our previous work considered classes in a metamodel as nodes, and relationships between classes are edges linking one node to another (Wu et al., 2013). Thus, we can formally define a graph, namely typed graph (*TG*) as: $TG = (V_T, E_T)$, where V_T and E_T represents a set of nodes (classes) or edges (associations and inheritances).

Each valid instance of a metamodel is also a graph: $G = (V_G, E_G)$, where V_G is set of nodes (objects) and E_G is set of edges (links), but preserve extra type information about the classes in a metamodel. Thus, we now can define a mapping between two graphs *TG* and *G*: $type = (type_V, type_E)$ where $type_V : V_G \rightarrow V_T$ and $type_E : E_G \rightarrow E_T$.

Now, we formally define a bounded typed graph as: $TG_b = (V_T, E_T, b)$ where b is a bound function $b : V_T \rightarrow Z^+$ that maps a typed node (non-abstract) to an integer. This integer specifies an upper bound of the same typed node that an instance may contain. Therefore, using this bound function, we can

bound our search space to guarantee the termination for metamodel instance generation.

For example, Figure 1 and 2 show a metamodel represented as a bounded typed graph and an instance of it. The bounded typed graph (metamodel) and graph (model) can be related with *type* such that $type_V(ComputerScience) = Department$, and $type_V(John) = type_V(William) = type_V(Robert) = Manager$. Similarly, $type_E(e1) = type_E(e2) = employ$.

To generate valid instances from a metamodel, we form a finite universe based on each bound defined for a typed node. This includes generating all possible links based on a particular association (edge) defined within the bound. We then use corresponding translation rules to encode them into SMT formulas. For example, for the typed graph shown in Figure 1, we form a finite universe containing 1 *Department* (*ComputerScience*) and 3 *Managers* (*John, William, Robert*). For the association *employ*, we form an adjacent matrix that describes all possible connections between *Department* and *Manager*. Each entry in the matrix is an SMT boolean variable indicating whether a link is selected or not. We then disjunct each entry in the matrix. The following steps show this basic translation for the metamodel in Figure 1 to the SMT formulas:

1. Form a finite universe:
 $\{ComputerScience, John, William, Robert\}$
2. For association *employ*, we form an adjacency matrix:

	<i>John</i>	<i>William</i>	<i>Robert</i>
<i>ComputerScience</i>	e_1	e_2	e_3

3. Generate SMT formula: $e_1 \vee e_2 \vee e_3$

The SMT formula captures the meaning of association *employ*: each *Department* is associated with at least one of the *Managers*. Figure 2 shows an example of only e_1, e_2 and e_3 are assigned to be *true* by an SMT solver, representing *John, William* and *Robert* are employed by the *ComputerScience* department.

In this paper, we assume all OCL constraints defined over a metamodel are not conflicted with both criteria. For example, a representative value of 18 is chosen for the attribute *age*, and an OCL constraint is defined as $self.age <> 18$.

3 Partition-based instance generation

The main idea for generating instances that satisfying *CA* and *AEM* coverage criteria is by adding

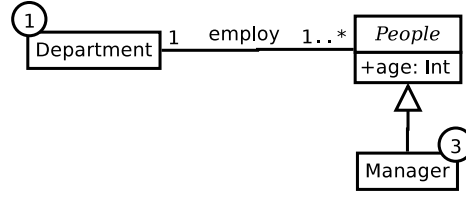


Figure 1: An example of a metamodel, represented as a bounded typed graph. The bounds for *Department* and *Manager* are depicted with a number in a circle on each class. In this case, they are 1 and 3 for *Department* and *Manager* respectively. No bound is specified for *People* as it is an abstract class.

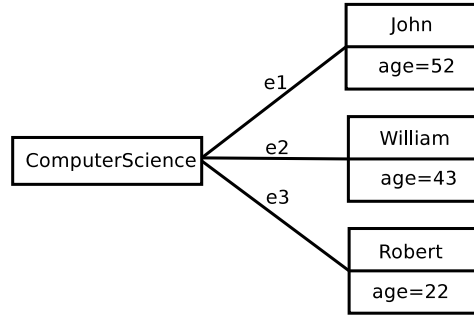


Figure 2: An instance of metamodel in Figure 1. This instance contains 1 instance of *department* and 3 instances of *manger*.

additional constraints expressed as SMT formulas to block irrelevant instances during the search. Each successful assignment is then interpreted as an instance that satisfies the coverage criteria. In the following sections, we show how these constraints can be expressed as SMT formulas.

For the set of features P in a metamodel, the general form of a constraint for each feature P_i in P can be expressed by:

$$\bigvee_{j=1}^{|P_i|} (T_i = V_j) \wedge F_i$$

where

- $|P_i|$ denotes the total number of partitions of a feature P_i .
- T_i is a *partition switch*, determines when a particular partition is to be switched on or off based on V_j .
- V_j indicates the j th partition of a feature P_i . This implies that the value for V_j chosen by an SMT solver determines which particular partition is selected.
- F_i is a *criteria formula* that is connected with a partition switch, indicating that when a partition switch is on the *criteria formula* must be applied.

- When an attribute d is an integer type:
 $((T_i = 0) \wedge (d < p)) \vee ((T_i = 1) \wedge (d = p))$
 $\vee ((T_i = 2) \wedge (d > p))$
- when an attribute d is a boolean type:
 $((T_i = 0) \wedge (d = false)) \vee ((T_i = 1) \wedge (d = true))$

Figure 3: SMT encoding for partitioning integer and boolean type attribute.

For different partition-based criteria, ensuring that the instances generated by the SMT solver achieve those criteria, depends on criteria formulas in each constraint. These criteria formulas are captured by the corresponding SMT encoding, and we elaborate these encodings in the Section 3.1 and 3.2.

3.1 Partitioning for class attributes

Achieving CA coverage requires that a constraint covers every partition created for each attribute, and this is controlled by criteria formula. In other words, the criteria formulas determine what value is to be assigned for an attribute in each instance.

Our current approach supports two types of attributes: integer and boolean, and the SMT encodings for those two types of attribute are presented in Figure 3.

As shown in Figure 3, for each i th attribute in a class, a partition switch (T_i) is created. For an integer type attribute, T_i has a value of 0, 1 or 2 indicating 3 partitions: $> p$, $= p$ and $< p$, where p is a representative value chosen by a user, each partition has a corresponding criteria formula. If no particular value is given, a value $p = 0$ will be chosen as default. These three partitions are directly expressed into SMT formulas. Similarly, an SMT encoding for a boolean type attribute is formed except that the partition switch is either 0 or 1, since a boolean value can only be *true* and *false*.

3.1.1 An Example of Attribute-based Partitions

As an example of how a criteria formula interacts with an attribute, we take the metamodel in Figure 4. In this metamodel the class called *Class* has an integer type attribute *methodCount* denoting the total number of methods contained in that class. The default strategy for an integer type attribute to achieve 100% is that it uses three partitions: $methodCount < 0$, $methodCount = 0$ and $methodCount > 0$. However, this would conflict with the OCL invariant which requires that *methodCount* must not be a negative number. Thus, with the default strategy only 66.6666%

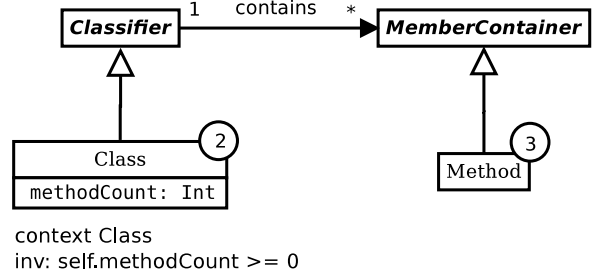


Figure 4: A subset of a programming language metamodel. The number in each circle represents the bound on the number of instances for a particular class (2 and 3 for *Class* and *Method* respectively).

can be achieved. This is because $methodCount < 0$ cannot be chosen by the SMT2 solver. In order to satisfy both OCL invariant and 100% coverage, we use a different strategy. We choose 3 as the value to partition *methodCount* into the three partitions: $methodCount < 3$, $methodCount = 3$ and $methodCount > 3$.

Since the bound for *Class* in Figure 4 is 2, we use *method1* and *method2* to capture attribute *methodCount*. Now a partition set $P = \{P_1, P_2\}$ is introduced where the elements P_1 and P_2 correspond to the data nodes *methodCount1* and *methodCount2*. Two partition switches are created, one each for P_1 and P_2 , ranging from 0 to 2 to represent the three possible partitions ($methodCount < 3$, $methodCount = 3$ and $methodCount > 3$). Each partition switch has a one-to-one mapping to a criteria formula. Each mapping is conjoined with a criteria formula which puts an extra constraint on each data node. Thus, the final formula generated is the conjunction of the partition switches for P_1 and P_2 , where

$$\begin{aligned}
 P_1 = & ((T_1 = 0) \wedge (methodCount1 < 3)) \\
 & \vee ((T_1 = 1) \wedge (methodCount1 = 3)) \\
 & \vee ((T_1 = 2) \wedge (methodCount1 > 3)). \\
 P_2 = & ((T_2 = 0) \wedge (methodCount2 < 3)) \\
 & \vee ((T_2 = 1) \wedge (methodCount2 = 3)) \\
 & \vee ((T_2 = 2) \wedge (methodCount2 > 3)).
 \end{aligned}$$

The disjunction inside P_1 and P_2 makes sure that at least one of the partitions is selected for each instance, and thus at least three instances must be found by the SMT2 solver to achieve 100% coverage for class attribute *methodCount*. Figure 5 shows the output generated by our tool in this case, depicting the three instances. These three instances show that the three partitions ($methodCount < 3$, $methodCount = 3$ and $methodCount > 3$) for the integer type attribute *methodCount* have been covered.

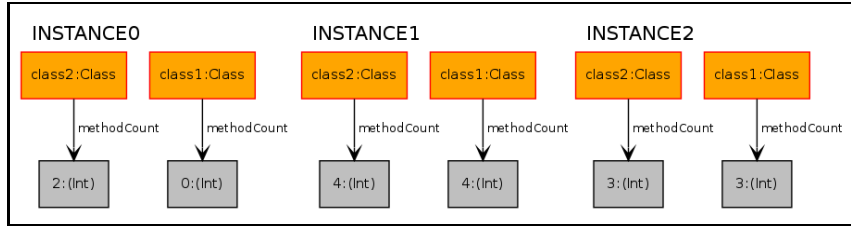


Figure 5: Three instances are needed to achieve a maximum *class attribute* coverage for the class *Class* in the metamodel from Figure 4.

3.2 Partitioning Associations

Associations between classes are an important part of a metamodel, and it is desirable that generated instances should also cover these associations for different partitions. The standard coverage criteria for associations, known as Association-end multiplicity (AEM), has already been defined in (Andrews et al., 2003), and in this section we show how this can be extended to metamodels and incorporated into our approach.

3.2.1 Partitioning Unidirectional Associations

To implement AEM coverage for unidirectional association, we specify *criteria formulas* corresponding to the most frequently used association types defined in a metamodel based on their multiplicities. These criteria formulas determine how each node (object) in an instance can be linked to others. For an association, we form all possible links from typed node to another based on the bound defined for each class at both association ends, and we then apply the corresponding translation rule to form a set of SMT formulas.

As shown in Section 2.2, we use an adjacent matrix E_{ref} to represent all possible links for an association *ref* between class *A* and *B*. The rows of the matrix, denoted as E_{row} , represent all links from one instance of *A* to one instance of *B*. The columns of the matrix, denoted as E_{col} , represent all links from one instance of *B* to one instance of *A*. Each entry $e_{i,j}$ in E_{ref} is an SMT boolean variable.

Figure 6 summarises 4 rules for common unidirectional association patterns. For each rule, there is a partition switch that is either 0 or 1 indicating that the association is divided into two partitions. For example, the second formula in Figure 6 shows an example of the translation rules for unidirectional association pattern $1..*$. This rule states that this association can be partitioned into two partitions, and each partition is controlled by a partition switch (T) and a criteria formula. One partition is that for each instance of *A* is associated with exactly one instance of *B*, and the other is for each instance of *A* is linked with a k number

$$Aux = \begin{array}{c|ccccc} & b_1 & b_2 & b_3 & b_4 & b_5 \\ \hline a_1 & 0 & 1 & 0 & 1 & 1 \\ a_2 & 1 & 0 & 1 & 0 & 1 \\ a_3 & 0 & 1 & 1 & 1 & 0 \end{array}$$

Figure 7: An example of a possible assignment found by an SMT solver for the auxiliary matrix.

of instances of *B*. In order to know the exact k number of instances of *B* that can be associated with an instance of *A*, both criteria formulas (associated with T) consist of an auxiliary matrix (Aux), where each element ($Aux_{i,j}$) in that matrix is an integer SMT variable. Each $Aux_{i,j}$ uses either 1 or 0 to denote whether a link in E_{ref} is selected or not. To compute k number of instances of *B* that connect to an instance of *A*, we add up all $Aux_{i,j}$ s in the same row to k . For example, Figure 7 shows a possible assignment found by an SMT solver. Each instance of *A* is connected to 3 instances of *B*, since each row in the array is added up to 3. Once an $Aux_{i,j}$ is chosen to be one, the corresponding $e_{i,j}$ in the matrix E_{ref} is also switched on (set to *true*). This indicates that a relevant link is presented in the instance.

3.2.2 Partitioning Bidirectional Associations

A bidirectional association distinguishes a unidirectional association by counting links in two directions. Therefore, a translation rule for a bidirectional association constrains both E_{row} and E_{col} . In general, the translation rules in Figure 8 are similar to unidirectional except that we need to correctly calculate the possible maximum number of instances of *B* that an instance of *A* connects to.

For example, the second rule in Figure 8 is for a bidirectional association pattern $1 \leftrightarrow 1..*^2$. This pattern is partitioned into two partitions. One is for each instance of *A* is connected to exactly one instance of *B*, and one instance of *B* can only connect to one instance of *A*. The other partition allows one instance of *A* to connect to multiple instances of *B*. Both parti-

²We use $x \leftrightarrow y$ to denote a bidirectional association with two multiplicities x and y at two association ends

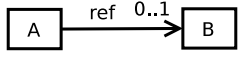
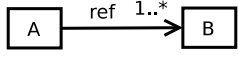
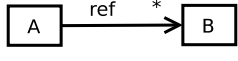
Association Pattern (Unidirectional)	Translation Rule
	$(1) \quad ((T = 0) \wedge \bigwedge_{i=1}^{ E_{row} } \bigvee_{j=1}^{ E_{col} } \neg e_{i,j})$ $\vee \quad ((T = 1) \wedge (\bigwedge_{i=1}^{ E_{row} } (\bigvee_{j=1}^{ E_{col} } (\bigwedge_{k=1, k \neq j}^{ E_{col} } \neg e_{i,k}) \wedge e_{i,j})))$
	$(2) \quad ((T = 0) \wedge \bigwedge_{i=1}^{ E_{row} } (\sum_{j=1}^{ E_{col} } Aux_{i,j} = 1))$ $\vee \quad ((T = 1) \wedge \bigwedge_{i=1}^{ E_{row} } (\sum_{j=1}^{ E_{col} } Aux_{i,j} = k \wedge E_{col} > 1))$ <p style="text-align: right;">where $1 < k \leq E_{col}$</p>
	$(3) \quad ((T = 0) \wedge \bigwedge_{i=1}^{ E_{row} } \bigvee_{j=1}^{ E_{col} } \neg e_{i,j})$ $\vee \quad ((T = 1) \wedge \bigwedge_{i=1}^{ E_{row} } (\sum_{j=1}^{ E_{col} } Aux_{i,j} = k \wedge E_{col} \geq 1))$ <p style="text-align: right;">where $1 \leq k \leq E_{col}$</p>

Figure 6: Translation rules for unidirectional association patterns.

tions are controlled by a partition switch (T), and have two different criteria formulas. The first criteria formula specifies the first partition is one instance of A connects exactly one instances of B , and one instances of B can only connect to one instances of A .

Since the second partition allows k number of instances of B to be connected to an instance of A , the criteria formula needs to compute the maximum possible number of instances of B that an instance of A can connect to. We compute this number k by calculating the difference between the bound of B and the bound of A , and adding 1. Thus, $|E_{row}|$ specifies $b(A)$ while $|E_{col}|$ gives $b(B)$. To understand how k gets calculated, we consider the following three scenarios:

1. $|E_{col}| = |E_{row}|$: we have equal number instances of A and B . Since the multiplicities for two association-ends (1 and 1..*) tell us that one instance of A must be connected to at least one instance of B , and one instance of B can only be linked to one instance of A , this scenario now implies that each instance of A connects each instance of B , vice versa. Thus, the maximum number of instances of B that an instance of A can connect to is $k = 1$.
2. $|E_{col}| > |E_{row}|$: we have more instances of B than A . We first connect every instance of A to one instance of B , and every instance of B connects to only one instance of A . Now, we can add the remaining number instances of B to one of the existing connections between instance of A and B , and counts one link as already having been estab-

lished. Thus, k gives the maximum number of B s that one of the instance of A can connect to.

3. $|E_{col}| < |E_{row}|$: we have less instances of B than A . However, this scenario violates the constraint implied by the multiplicities, and is thus ruled out.

In all possible cases the minimum number of instances of B has to be equal to the number instances of A . Figure 9 shows an example where one instance of A can connect to at most 3 (here we set $k = 3$) instances of B .

Similarly, rule 1 in Figure 8 states that when the first partition is chosen (T is 0), no links encoded by $e_{i,j}$ are chosen. When the second partition is chosen (T is 1), only one $e_{i,j}$ is chosen. This indicates that a link ($e_{i,j}$) between each instance of A and B is allowed to be presented. Rule 3 for the association pattern ($1 \leftrightarrow *$) has a criteria formula that combines the first criteria formula from the first association-pattern ($1 \leftrightarrow 0..1$) with the second criteria formula from the second association-pattern ($1 \leftrightarrow 1..*$) as they indicate that either no links are chosen at all or choose exactly k number of links between an instance of A and B .

3.3 Instance Enumeration

With respect to the coverage criteria presented in previous sections, we devise a way to reduce the number of instances during the enumeration. We first group partition switches that have the same number of partitions, and then apply Formula 1 to block unnecessary assignments. To group them together, we store ev-

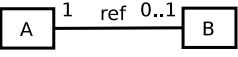
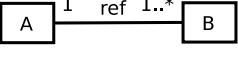
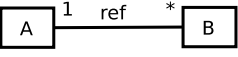
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	$(2) \quad ((T = 0) \wedge \bigvee_{i=1}^{ E_{row} } (\sum_{j=1}^{ E_{col} } Aux_{i,j}) = 1)$ $\vee \quad ((T = 1) \wedge \bigvee_{i=1}^{ E_{row} } (\sum_{j=1}^{ E_{col} } Aux_{i,j}) = k) \wedge E_{row} < E_{col} $ <p style="text-align: center;">where $1 < k \leq E_{col} - E_{row} + 1$</p>
	$(3) \quad ((T = 0) \wedge \bigwedge_{i=1}^{ E_{row} } \bigvee_{j=1}^{ E_{col} } \neg e_{i,j})$ $\vee \quad ((T = 1) \wedge \bigvee_{i=1}^{ E_{row} } (\sum_{j=1}^{ E_{col} } Aux_{i,j}) = k) \wedge E_{row} \leq E_{col} $ <p style="text-align: center;">where $1 \leq k \leq E_{col} - E_{row} + 1$</p>

Figure 8: Translation rules for bidirectional association patterns.

	b_1	b_2	b_3	b_4	b_5
$Aux = a_1$	1	0	0	1	1
a_2	0	0	1	0	0
a_3	0	1	0	0	0

Figure 9: An example of an assignment found by an SMT solver for an association between two classes A and B , where the bounds for A and B are 3 and 5, respectively. In this example an instance of A is linked with a maximum of 3 instances of B . Since the sum of each column is 1, an instance of B is only connected to a single instance of A .

every partition switch in a hash table, indexed by their number of partitions. Then we apply Formula 1 on each group of switches. The first sub-formula in Formula 1 indicates that every partition switch that has the same number of partitions must be equal to each other. This allows us to achieve the coverage criteria partitions by partitions. The second sub-formula indicates that it is also possible for switches that have the same number of partitions to have different combinations. This allows us to achieve the coverage criteria using a combination of partitions. Finally, we conjoin all the formulas from each group to have a combination of partition switches that have different partitions.

$$\left(\bigwedge_{i=1}^{n-1} T_i = T_{i+1} \right) \vee \left(\bigwedge_{i=1}^{n-1} T_i \neq T_{i+1} \right), \text{ where } n > 2. \quad (1)$$

To understand how this works, please consider the following example:

Suppose a total of six partition switches

2	T_1, T_2, T_3, T_4
3	T_5, T_6

Table 1: Partition switches are grouped by the number of partitions

($T_1, T_2, T_3, T_4, T_5, T_6$) were created, four of them (T_1, T_2, T_3, T_4) have two partitions and two of them (T_5, T_6) have three partitions. We first group those six partition switches, as in Table 1, by their partitions. Then we apply Formula 1 to these two groups. This results in the following formulas:

- $f_1 = ((T_1 = T_2) \wedge (T_2 = T_3) \wedge (T_3 = T_4)) \vee ((T_1 \neq T_2) \wedge (T_2 \neq T_3) \wedge (T_3 \neq T_4))$
- $f_2 = (T_5 = T_6) \vee (T_5 \neq T_6)$ ³

We observe these formulas, not every assignment that makes f_1 and f_2 *satisfiable* is possible now. For example, T_1, T_2, T_3, T_4 can have a total of sixteen possible assignments ($T_1 \vee T_2 \vee T_3 \vee T_4$). However, after applying Formula 1, only four possible assignments are left. These four possible assignments are listed in Table 3. The first two assignments satisfy the first sub-formula of Formula 1, and the remaining two satisfy the second sub-formula. Thus, by applying Formula 1, we block twelve other possible assignments. Furthermore, the first two assignments achieve full coverage via partitions. The last two assignments achieve full coverage via a combination of partitions. Therefore, by enumerating these four pos-

³Note that when there are only two partition switches (T_i and T_j), we use formula $(T_i = T_j) \vee (T_i \neq T_j)$.

T_1	T_2	T_3	T_4
1	1	1	1
2	2	2	2
1	2	1	2
2	1	2	1

Table 2: Four possible assignments for formula f_1 . Here, 1 and 2 denote two different partitions.

T_1	T_2	T_3	T_4	T_5	T_6
1	2	1	2	3	2

Table 3: One possible assignment for formula f_1 and f_2 . Here, 1, 2 and 3 denote three different partitions.

sible assignments, we can cover all the partitions for partition switches (T_1, T_2, T_3 and T_4) in two different ways.

Finally, in order to have a different combination from partition switches that have different partitions, we conjoin the two formulas ($f_1 \wedge f_2$) and feed the entire formula to an SMT2 solver. For example, Table 3 shows one possible assignment for all partition switches ($T_1, T_2, T_3, T_4, T_5, T_6$).

3.4 An Example of Achieving CA and AEM Coverage Criteria

To demonstrate partition-based generation, we use a subset of the Ecore metamodel as depicted in Figure 10 (Steinberg et al., 2008). This metamodel describes the relationships among *EPackage*, *EClass*, *EAttribute*, and *EOperation* in the Ecore metamodel. The bound for each non-abstract class is shown as a number in a circle.

In order to achieve the maximum coverage for both CA and AEM, the translation rules described in Section 3 are applied to this metamodel, and result in a total of 8 instances. Thus, one can conclude that only 8 instances are needed to achieve the full coverage of CA and AEM for this metamodel. These 8 instances cover a range of combinations from different partition switches defined during the translation to SMT2 formulas. For example, two of the instances in Figure 11 and 12 show a combination of different partitions from different associations and attributes defined in the metamodel.

4 Instance generation using graph-based constraints

The techniques described in the previous sections allow coverage criteria to be achieved for the class attributes and associations in a metamodel. As such

they are standard coverage criteria for UML class diagrams, and may be applied to any metamodel. However, it is reasonable to suppose that a user will have more sophisticated requirements, and wish to direct the generation of instances so as to satisfy other constraints. For this reason, we present a new technique that allows instance generation to meet the following graph-based properties.

4.1 Directed Acyclic Graphs

Directed acyclic graphs (DAGs) are commonly used in many areas, for example, the topology of a network, data flow diagrams, etc. Regarding metamodeling, one may require a program to have a particular depth of inheritance tree, or a particular call depth. Thus, to ensure the generation of a DAG from a reflexive association in a metamodel, we only enable the elements that are in the upper triangle of the adjacency matrix, and disable the rest of the $e_{i,j}$ s in the matrix, breaking all the cycles in the graph.

4.2 Sharing and Non-Sharing Nodes

Since we use an adjacency matrix to capture all possible links (within the bounds) for an association, we can also manipulate this matrix to form new formulas that can express how links are connected to each other. In particular, we can facilitate the specification of *graph-based constraints* by the user that will direct instance generation. In order to facilitate instance generation with such properties, we introduce the following new properties.

In a graph, some nodes may have all their outgoing edges going to the same node and some may not. We consider these nodes having sharing and non-sharing properties. Sharing and non-sharing properties can only be applied to a *non-reflexive* association. Before we precisely define sharing and non-sharing properties, we first define two functions (f and g).

1. Function f is an out-adjacency function (Adj^+) that computes a set of nodes from all out-going edges of a particular node. $f : V_G \rightarrow 2^{V_G}$, where V_G is the set of nodes, and 2^{V_G} is the power set of V_G .
2. Function g is an in-adjacency function (Adj^-) that computes a set of nodes from all in-coming edges of a particular node. $g : V_G \rightarrow 2^{V_G}$, where V_G is the set of nodes, and 2^{V_G} is the power set of V_G .

With functions f and g , we are able to calculate a set of nodes based on their in-coming and out-going edges. Now we can use these two functions to define the following sharing and non-sharing properties:

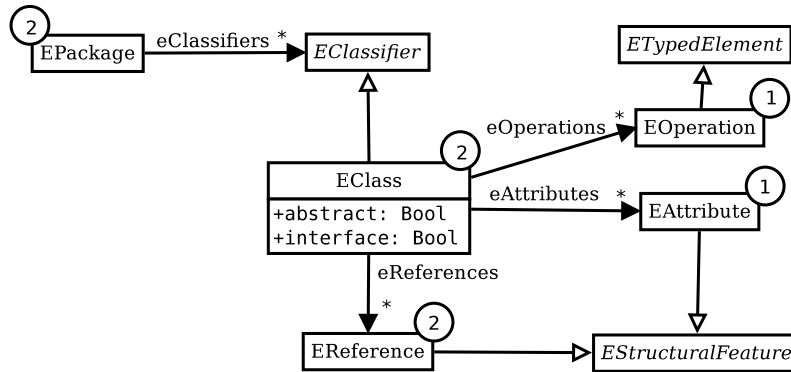


Figure 10: A subset of the Ecore metamodel showing the relationship between *EPackage*, *EClass*, *EAttribute*, *EReference* and *EOperation*, where a bound for each non-abstract class is depicted as a number in a circle.

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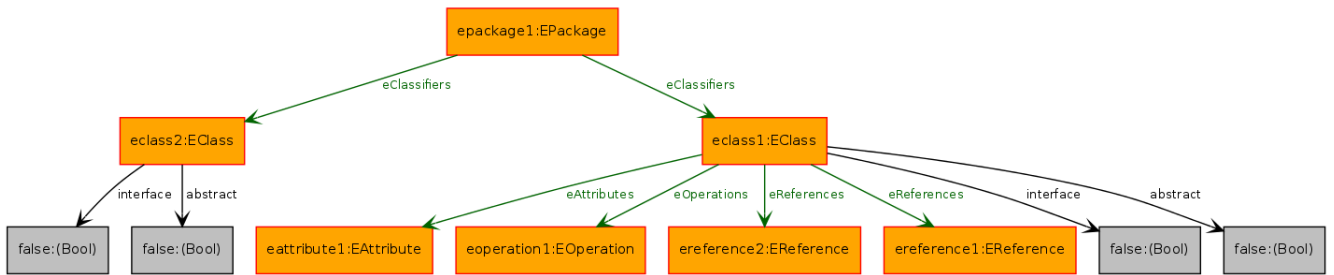


Figure 11: In this generated instance of the Ecore metamodel of Figure 10, at least one *EClass* instance is associated with a maximum number of *EAttribute*, *EOperation* and *EReference* instances.

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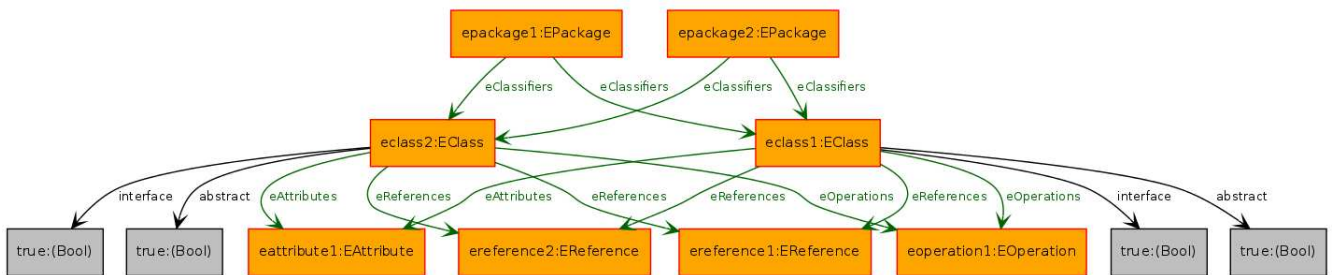


Figure 12: In this generated instance of the Ecore metamodel of Figure 10, each *EClass* instance is associated with a maximum number of other instances (*EAttribute*, *EOperation* and *EReference*) according to the bound defined on each class in metamodel.

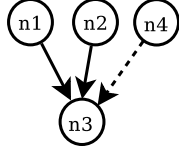


Figure 13: An example of sharing nodes in a graph

- A set of nodes $L = \{N_1, N_2, \dots, N_j\}$, where $|L| \geq 2$, are said to be *strong sharing nodes* iff $(\bigcap_{i=1}^j f(N_i)) \neq \emptyset, \forall L_x \in \bigcup_{i=1}^j f(N_i)$ and $g(L_x) \subseteq L$.
- A set of nodes $L = \{N_1, N_2, \dots, N_j\}$, where $|L| \geq 2$, are said to be *weak sharing nodes* iff $(\bigcap_{i=1}^j f(N_i)) \neq \emptyset, \exists L_x \in \bigcup_{i=1}^j f(N_i)$ and $L \subset g(L_x)$.
- A set of nodes $L = \{N_1, N_2, \dots, N_j\}$, where $|L| \geq 2$, are said to be *strong non-sharing nodes* iff $\forall L_i \in L, |f(N_i)| = 1$ and $f(N_a) \cap f(N_b) = \emptyset$, where $1 \leq a < b \leq j$.
- A set of nodes $L = \{N_1, N_2, \dots, N_j\}$, where $|L| \geq 2$, are said to be *weak non-sharing nodes* iff $\forall N_i \in L, |f(N_i)| > 1$ and $f(N_a) \cap f(N_b) = \emptyset$, where $1 \leq a < b \leq j$.

To understand these definitions, we use two examples to illustrate sharing and non-sharing properties. In Figure 13, a solid line is used to denote the existing links and a dashed line is used to represent possible links. The set of nodes $n1$ and $n2$ (with solid lines) are considered as strong sharing nodes since both their out-adjacency functions return $n3$ ($f(n1) = f(n2) = \{n3\}$), and $n3$'s in-adjacency function returns $n1$ and $n2$ ($g(n3) = \{n1, n2\}$). In other words, $n3$ can only be accessed by both $n1$ and $n2$ and no other nodes. However, if a link from $n4$ to $n3$ is connected, then the set of nodes $n1$ and $n2$ are regarded as weak sharing nodes because $n3$'s in-adjacency function this time returns three nodes: $g(n3) = \{n1, n2, n4\}$. Thus, the set of nodes $n1, n2$ and $n4$ are considered as strong sharing nodes ($f(n1) \cap f(n2) \cap f(n4) = n3$), and $g(n3) \subseteq \{n1, n2, n4\}$.

Similarly, in Figure 14 the solid lines between nodes $n1, n2$ and $n4, n5$ make the set of nodes $n1$ and $n4$ strong non-sharing nodes in the graph ($|f(n1)| = |f(n4)| = 1$, and $f(n1) \cap f(n4) = \emptyset$). If $n1$ also connects to $n3$ (a possible link), and $n4$ connects to $n6$, then the set of nodes $n1$ and $n2$ are weak non-sharing nodes, since they all connect to more than one other node ($|f(n1)| = |f(n4)| > 1$).

Figure 16 shows a matrix for capturing an association in a metamodel. Suppose we want to give strong sharing property to a set of nodes $L = \{a_1, a_4\}$.

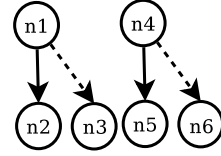


Figure 14: An example of non-sharing nodes in a graph

	a_1	a_2	a_3	a_4
b_1	$e_{1,1}$	$e_{1,2}$	$e_{1,3}$	$e_{1,4}$
b_2	$e_{2,1}$	$e_{2,2}$	$e_{2,3}$	$e_{2,4}$
b_3	$e_{3,1}$	$e_{3,2}$	$e_{3,3}$	$e_{3,4}$

Figure 16: An example of matrix for illustrating sharing and non-sharing properties. In this example, each $e_{i,j}$ is represented as a link from a_j to b_i .

This indicates that at least one of the b 's must be shared by them. For example, $e_{1,1}$ and $e_{1,4}$ could be selected at the same time, or $e_{2,1}$ and $e_{2,4}$ are chosen ($(e_{1,1} \wedge e_{1,4}) \vee (e_{2,1} \wedge e_{2,4})$). This represents that a_1 and a_4 they both have out-going edges to b_1 or b_2 . This is captured by the first sub-formula of rule (2) from Figure 15. Now, suppose $e_{1,1}$ and $e_{1,4}$ are selected, then anything between them cannot be selected otherwise they are not strong sharing nodes. Thus, $e_{1,2}$ and $e_{1,3}$ are disabled when $e_{1,1}$ and $e_{1,4}$ are selected ($(e_{1,1} \wedge e_{1,4}) \rightarrow (\neg e_{1,2} \wedge \neg e_{1,3})$). This is captured by the second sub-formula of rule (2) from Figure 15.

Similarly weak sharing, strong and weak non-sharing properties are captured in rule (1) (3) and (4) in Figure 15. In each formula listed in Figure 15, we use L to denote a set of nodes to be assigned with one of the four properties, and $|L| \geq 2$. For weak sharing property, the Formula is similar to the Formula for strong sharing property except that we drop the second sub-formula. Instead, we add a formula that states that at least one of the a 's not specified in L can be linked to the b 's. The formula for the strong non-sharing property indicates that only one link can be selected according to specified nodes in L . It indicates that as long as one link is selected all other links in the same row and column are switched off. Similarly, for weak non-sharing property, the formula indicates that there could be multiple links selected according to a specific node in L . This indicates that a node can connect to at least one or more nodes. Since connections to multiple nodes are allowed, all other nodes in the same row must be disabled.

4.3 An Example of Achieving Graph-based Constraints

This section demonstrates an example of achieving graphs-based constraints via using the techniques de-

Property	SMT Formula
(1) Weak sharing	$\bigvee_{i=1}^{ E_{row} } \bigwedge_{k=1}^{ L } e_{i,L_k} \wedge \bigwedge_{i=1}^{ E_{row} } \bigvee_{j=1, j \notin L}^{ E_{col} } e_{i,j}$
(2) Strong sharing	$(\bigvee_{i=1}^{ E_{row} } \bigwedge_{k=1}^{ L } e_{i,L_k}) \wedge (\bigwedge_{i=1}^{ E_{row} } (\bigwedge_{k=1}^{ L } e_{i,L_k} \rightarrow \bigwedge_{j=1, j \notin L}^{ E_{col} } \neg e_{i,j}))$
(3) Strong non-sharing	$(\bigwedge_{k=1}^{ L } \bigvee_{i=1}^{ E_{row} } (\bigwedge_{j=1, j \neq i}^{ E_{row} } \neg e_{j,k}) \wedge e_{i,L_k}) \wedge (\bigwedge_{i=1}^{ E_{row} } (\bigwedge_{k=1}^{ L } e_{i,L_k} \rightarrow \bigwedge_{j=1, j \notin L}^{ E_{col} } \neg e_{i,j}))$
(4) Weak non-sharing	$(\bigwedge_{k=1}^{ L } \bigvee_{i=1}^{ E_{row} } e_{i,L_k}) \wedge (\bigwedge_{i=1}^{ E_{row} } (\bigwedge_{k=1}^{ L } e_{i,L_k} \rightarrow \bigwedge_{j=1, j \notin L}^{ E_{col} } \neg e_{i,j}))$

Figure 15: The SMT formulas for capturing sharing/non-sharing properties.

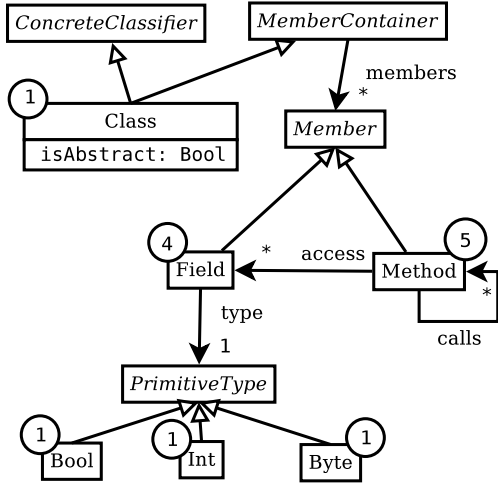


Figure 17: A subset of the Java metamodel represented as a bounded graph representing relationships between classes and their members, which are fields or methods.

scribed in Section 4. This example uses a small subset of Java programming language metamodel as depicted in Figure 17. This metamodel represents the relationship (a field can be *accessed* by multiple methods, and a method can *call* multiple methods) between *Class*, *Method* and *Field*. From this metamodel, instances of programs with a particular value of a cohesion metric (LCOM) can be generated (Chidamber and Kemerer, 1994), and with a particular depth of the call graph (Li and Henry, 1993).

LCOM is a structural class cohesion metric that measures the number of disjoint components in a graph, where each node represents a method and an edge indicates that two methods share at least one common field. In order to generate instances, an SMT2 solver is used to compute how the graph should be connected according to the bounds defined over the metamodel.

Thus, an SMT2 solver is used for finding assignments for the constraints encoded in Formula 2:

$$\left(\bigwedge_{k=1}^c \bigvee_{j=1}^{|Method|} m_j = k \right) \wedge \left(\bigwedge_{j=1}^{|Method|} 1 \leq m_j \leq c \right) \quad (2)$$

Here, c is the desired value for the LCOM metric specified by the user, for example LCOM should be evaluated to 3 that means a graph has 3 connected components. Thus, c denotes the number of connected components. We use an SMT2 integer variable, m_j to encode a method. The possible values of an m_j can get indicates that whether the corresponding method is connecting to another.

If two methods are assigned the same integer, this indicates that they are connected in the graph (one connected component), otherwise they are disconnected. Note that we require that c varies between 1 and the number of methods bounded by $|Method|$ inclusive, that is because the connection of a list of *methods* varies from not being connected at all to being fully connected.

To understand how Formula 2 works, we use the following example.

Suppose we have five *methods* (*method1*, *method2*, *method3*, *method4*, and *method5*), and we would like them to form a graph with three connected components. That means three different integer values should be assigned to those *methods* to indicate three different connected components ($c = 3$). Any two *methods* get assigned to the same integer means that they are connected. Now, we expand Formula 2, we get the following terms:

$$\begin{aligned} & ((m_1 = 1) \vee (m_2 = 1) \vee (m_3 = 1) \vee \\ & (m_4 = 1) \vee (m_5 = 1)) \wedge \\ & ((m_1 = 2) \vee (m_2 = 2) \vee (m_3 = 2) \vee \\ & (m_4 = 2) \vee (m_5 = 2)) \wedge \\ & ((m_1 = 3) \vee (m_2 = 3) \vee (m_3 = 3) \vee \\ & (m_4 = 3) \vee (m_5 = 3)) \wedge \\ & (1 \leq m_1 \leq 3) \wedge (1 \leq m_2 \leq 3) \wedge (1 \leq m_3 \leq 3) \wedge \\ & (1 \leq m_4 \leq 3) \wedge (1 \leq m_5 \leq 3) \end{aligned}$$

In the expanded formulas above each m_j denotes a *method*, for example m_1 denotes *method1*. Now ob-

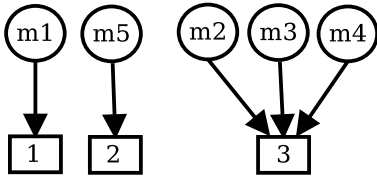


Figure 18: One of the successful assignments for the formula for deciding how the graph is connected, based on the bounds defined on metamodel in Figure 17 and given a desired *LCOM* value of 3. Here m_1 through m_5 represent methods, and the numbers 1 through 3 represent three sets in a partition of these methods.

serve these formulas, we note that three of the m'_j s can be assigned with value of 1,2 and 3. Because of the additional constraints $1 \leq m_j \leq 3$, this guarantees that the remaining two m'_j s share some integers with other m'_j s, since there are only 3 possible integer values to be assigned to 5 methods.

One possible solution (Note that one might get a different solution, this depends on the SMT solver.) derived from Formula 2 is shown in Figure 18, where m_2 , m_3 and m_4 are assigned with the same value of 3, indicating that *method2*, *method3* and *method4* are connected together in the graph. That means that these three methods share at least one common field. On the other hand, m_1 and m_5 are assigned values that are different from others. This specifies that *method1* and *method5* are disjoint from other methods. Therefore, a graph with an *LCOM* value of 3 has been constructed, and we can enumerate each successful assignment for this formula to get every possible graph with this *LCOM* value.

Now that how the graph is structured for an *LCOM* value of 3 is known, sharing and non-sharing formulas on the five methods can be applied. More specifically, the column can be located from the 2D-array capturing the *access* association, and two lists are constructed, one each for the sharing and non-sharing nodes. In this example, we simply use weak sharing formulas on *method2*, *method3* and *method4*, and strong non-sharing formulas on *method1* and *method5*. As Figure 19 shows, *method2*, *method3* and *method4* all have access to *field3*, while *method1* and *method5* have accesses to *field2* and *field4* respectively.

To generate the call graph with a depth of 3 for *Method* in the metamodel in Figure 17, we generate a directed acyclic graph (Section 4.1) for the association *calls*. Figure 19 shows the series of method calls giving a depth of 3: *method1* calls *method4* and *method4* calls *method5* which calls *method3*.

5 Evaluation

In this section, we first briefly describe a tool that is extended with partition-based and graph-based instance generation, then we present our initial evaluation, and finally we discuss its capabilities and limitations.

5.1 Implementation

We have implemented and integrated the partition-based and graph-based criteria described in this paper in our existing tool, ASMIG³. ASMIG takes in a metamodel in ecore format (with a bound defined for each class in that metamodel) and an OCL file, outputs all consistent models (if it has any) within those bounds. To enumerate “all possible” instances, ASMIG blocks all previously generated instances by adding the negation of satisfiable assignments found by an SMT solver one at a time until no more satisfiable assignment is possible. ASMIG is purely written in Java and it consists of about 22000 (excluding UI) lines of code (LOC) with about 8300 LOC dedicated to the techniques described in this paper. To construct ASMIG, we re-engineered a parser extracted from the USE tool (Kuhlmann et al., 2011), adapting it to use as our front-end for reading the OCL invariants. The current version of ASMIG uses Z3 as its default back-end SMT solver (De Moura and Bjørner, 2008), supports generating formulas in SMT2 standard (Barrett et al., 2010).

5.2 Results

The evaluation for both partition-based and graph-based criteria is performed on a machine with a 2.93GHz Intel Core 2 Duo and 4GB memory, the results and time are recorded in Table 4 and 5. More detailed results, along with further examples, are available at our website⁸.

Table 4 shows the results for partition-based criteria based on a total of 20 metamodels. For each metamodel, we record translation time and average instance generation time. These recorded time along with the size of metamodels can be seen in Figure 20. From Table 4 and Figure 20, we note that the translation time is not affected by a single factor. In fact,

³available at: <https://github.com/NUIM-FM/asmig>

⁴Available at: <http://www.emn.fr/z-info/atlanmod/>

⁵From Eclipse Modeling Framework Royal and Loyal Example Project

⁶Extracted from Eclipse Modeling Framework

⁷Available at: <http://www.jamopp.org/>

⁸<http://www.cs.nuim.ie/~haowu/ASMIG/Results/>

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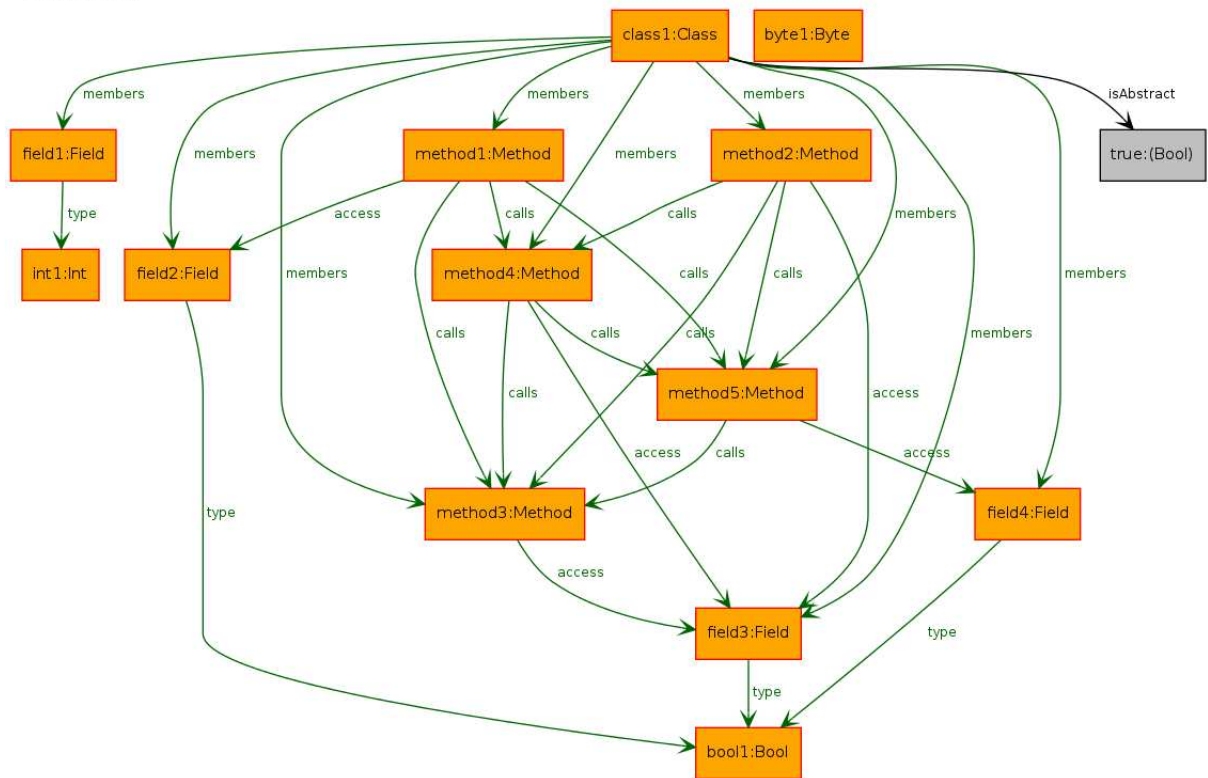


Figure 19: A generated instance of the Java metamodel from Figure 17. This Java program has an *LCOM* value of 3, as well as a call-graph depth of 3.

it is affected by three factors: the size of a metamodel, the number of associations and the number of attributes. A smaller metamodel with more associations and attributes, ASMIG might need more time for translation than a larger metamodel with less associations and attributes. This is because each association and attribute needs to be additionally constrained in order to achieve partition-based coverage criterion. For example, the metamodel *HTML* has more classes than the *UML2 Class Diagram* metamodel, but ASMIG spends much more time on translating the *UML2 Class Diagram* metamodel than *HTML* because the *UML2 Class Diagram* has 26 associations and 46 attributes while *HTML* only has 7 associations and no attributes. Therefore, translation time from a metamodel to formulas depends on its size, associations and attributes.

To evaluate scalability, we select a variety of metamodels such as general purpose programming languages, domain specific languages, ranging from small size to large size. We believe these metamodels are good representatives in terms of their usage in different domains. To effectively evaluate partition-based technique, we change ASMIG’s internal configuration in order to set a large enough upper bound for each non-abstract class, this would guarantee each non-abstract class to be initialised at least once and up to that upper bound. The *total bounds* column in Figure 4 shows the sum of each bound for every non-abstract class in the metamodel. The *total instances* column indicates the number of instances generated in order to achieve full coverage for the CA and AEM criteria. For the class attributes coverage criteria (*CA*), ASMIG allows users to choose a representative value, but for general purpose, we choose the default value 0 to obtain three partitions (< 0 , $= 0$ and > 0) for each integer type attribute. For the association-end multiplicity criteria (*AEM*), we set 3 instances as an upper bound for each association that has a multiplicity of *. We choose this upper bound because it is easy for us to distinguish this from a one-to-one multiplicity for both association ends.

For graph-based instance generation, we have evaluated ASMIG against different bounds for each metric to measure its scalability. The graph in Figure 21 shows that the translation time is affected by the bounds defined. In general, the translation and average finding time is proportional to the size of bounds allocated on both association-ends. However, ASMIG tries to utilise cache mechanism as much as possible to prevent formula regeneration, and this depends on a specific association defined in a metamodel.

Table 5 shows the results for ASMIG to gener-

CK Metric	Metric Value	Total Bound	Time in ms	
			Translate	Finding
WMC	2	3	446	31
	3	6	444	59
	5	10	461	120
DIT	2	3	456	53
	4	6	457	54
	8	10	460	180
NOC	2	3	469	64
	4	6	481	65
	8	10	489	180
LCOM	2	3	476	63
	2	6	470	42
	3	10	484	138

Table 5: Results of generating 100 instances which satisfy constraints based on four of the CK metrics. Each metric was constrained using three values, and the calculated bounds are shown, as well as two measures of the time taken to generate appropriate instances.

ate 100 instances of a subset of Java programming language metamodel based on specific bounds for four different CK metrics (Chidamber and Kemerer, 1994). We choose this metamodel because the graph-based criteria focus on a specific association such as an inheritance relationship must be a directed acyclic graph. We choose CK metrics because these metrics possess good graph properties.

For simplicity, we consider the complexity of WMC as the sum of the methods in a class. To correctly calculate LCOM value, we first use an SMT solver to select a list of nodes to be connected, then apply sharing/non-sharing properties to those nodes. There are a number of possible definitions of the LCOM metric which are supported by ASMIG. We choose LCOM3 here which represents the methods accessing common fields as a connected graph. Thus, a value of 2 for LCOM3 means 2 connected graphs with respect to method accessing fields. For each of the four metrics in Table 5 we specified three different metric values (shown in the *values* column) causing the calculated bounds to vary between 3 to 10. For DIT, we directly apply the method described in Section 4.1. For NOC, we first fix the number of links between two nodes via an auxiliary matrix as shown in Section 3.2, then apply strong sharing properties to these nodes. All the detailed instances generated for above metrics are available at our website ⁸.

5.3 Discussion

Capabilities. (1) The partition-based technique in section 3.1 and section 3.2 allows one to achieve a full equivalence partitioning testing by iteratively se-

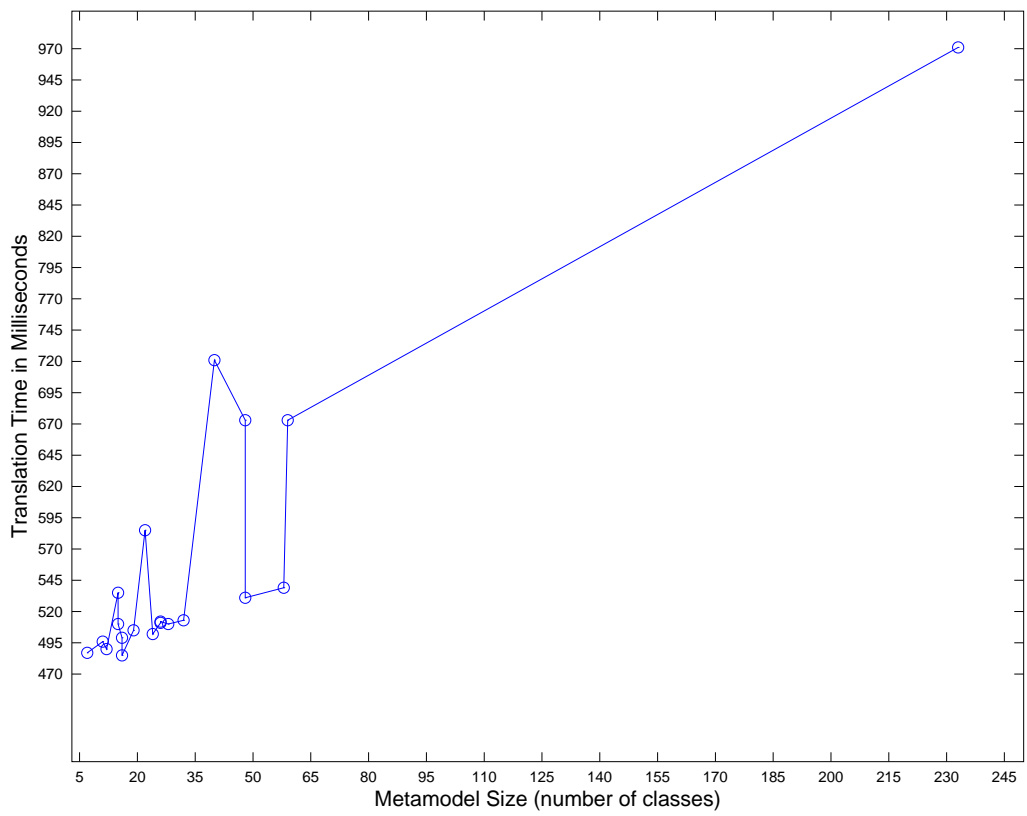


Figure 20: Translation time affected by different sized metamodel in Table 4. Each point in this graph represents the translation time that ASMIG takes on a specific metamodel from Table 4.

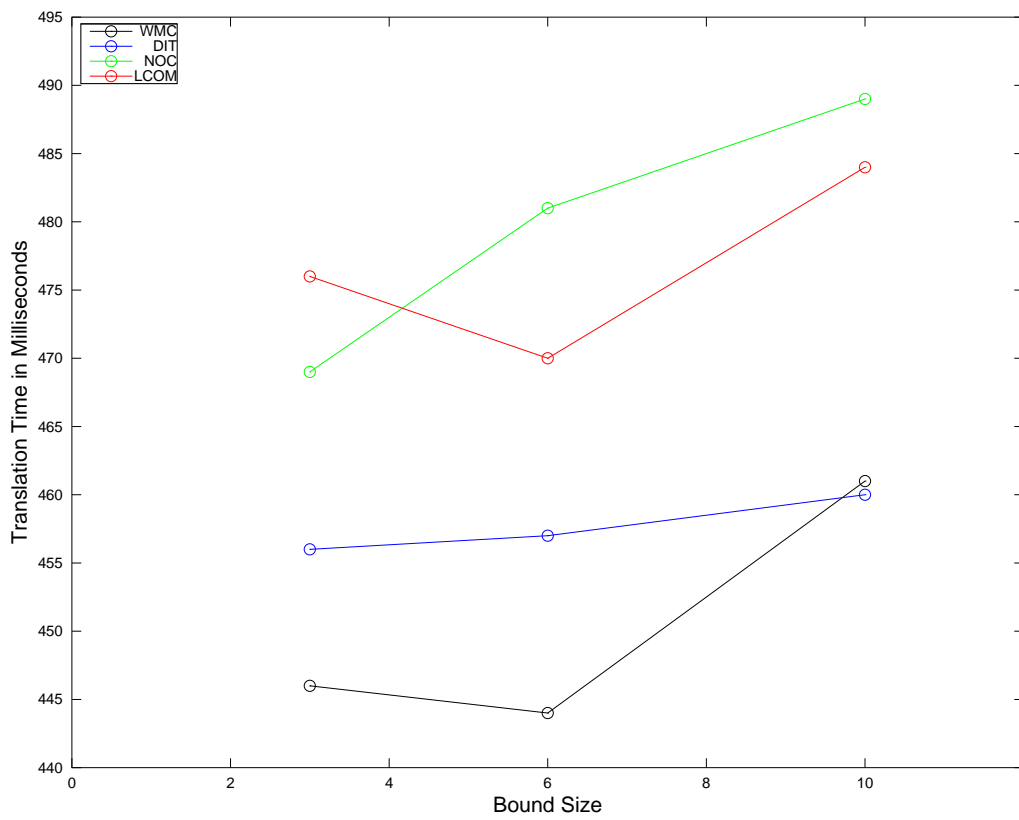


Figure 21: Translation time against different size of bound for four metrics. Each point in this graph represents one specific translation time that ASMIG takes based on a specific bound. Points and bounds are derived from Table 5.

Metamodel	Number of			Total		Time in ms	
	Classes	Assocs	Attribs	Bounds	Instances	Translation	Avg Finding
Ant ⁵	48	27	0	56	4	673ms	71ms
Company ⁶	7	6	6	13	12	487ms	36ms
C++ 1.0 ⁵	16	4	5	20	8	499ms	22ms
Java ⁹	233	104	1	183	8	971ms	2629ms
Royal&Loyal ⁷	15	41	2	40	8	535ms	65ms
Finite State Machine 1.0 ⁵	16	7	0	16	4	485ms	30ms
Ecore ⁸	22	40	0	31	4	585ms	195ms
UML2 Class Diagram ⁸	40	26	46	35	8	721ms	966ms
Web App: Conceptual Model ⁵	19	24	0	25	4	505ms	45ms
KM3 ⁵	12	7	0	16	4	490ms	40ms
Business Process Model ⁵	26	15	0	28	4	511ms	63ms
CPL1.0 ⁵	32	16	0	38	4	513ms	90ms
GraphML ⁵	11	13	2	20	8	496ms	53ms
Hierarchical State Machine 1.0 ⁵	15	16	0	33	4	510ms	72ms
Maven(maven.xml) 0.3 ⁵	58	32	0	65	4	539ms	108ms
MoDAF0.1 ⁵	48	35	0	70	4	531ms	76ms
QualityofService ⁵	24	26	0	37	4	502ms	69ms
DOT1.0 ⁵	26	20	0	21	4	512ms	50ms
BibTexML1.2 ⁵	28	4	0	18	4	510ms	28ms
HTML ⁵	59	7	0	59	4	673ms	1562ms

Table 4: Results of 20 metamodels for evaluating partition-based instance generation, and all instances were automatically generated by the ASMIG tool.

lecting a different representative value. Equivalence partitioning testing is considered as one of the important techniques for testing object oriented system (Binder, 1999) (Gutjahr, 1999). For *CA*, equivalence partitioning testing for a valid range of 0..100 can be achieved via two steps. Firstly, pick 0 as the representative value covering < 0 , $= 0$ and > 0 , then choose 100 covering < 100 , $= 100$ and > 100 . Similarly, for *AEM*, one can set a larger bound just outside the boundary (the boundary can be decided by using knowledge of the problem domain) for each class at two ends of an association. (2) Having instances meeting graph-based constraints provides a way of analysing or measuring a software system such as generating a control flow graph via specifying sharing/non-sharing properties on specific nodes. Viewing a metamodel or an instance as a graph brings one kind of diversity of instance generation for those who require models that are based on particular shape of a graph.

Limitations. (1) Currently, our SMT formulas do not

fully support a graph criteria that constrains over more than a single association. For example, metrics like response for a class (RFC) or coupling between object classes (CBO) typically constrain over two different associations. However, this can be avoided via a sequence of SMT solving, and use the assignment from previous successful solving as the input to the next SMT solving. For example, for RFC, one could fix a set of methods first, then use SMT solver to distribute the number of methods directly called by that set of methods, finally apply sharing/non-sharing properties to those methods are selected from previous SMT solving. (2) We admit that both partition-based and graph-based criteria may not be sufficient enough to fulfill users' expectation of the diversity of instances. One may certainly require a different criteria for validating a metamodel based on different testing strategies. *CA* and *AEM* criteria both are part of standard coverage criteria for UML class diagrams, and graph-based criteria provides a way of generating instances based on describing graph properties. With

both kinds of instance generation, we can at least provide a certain degree of confidence in designing, testing or validating a metamodel. In the future, we will investigate a more general approach that would allow language engineers to describe customised coverage criteria via a simple domain specific language.

6 Related work

One of the challenges with metamodelling is that it is difficult to instantiate a metamodel since instances have to conform to both the metamodels' structural constraints and additional semantic constraints written in a language such as OCL. Although much recent research has endeavoured to instantiate metamodels using different approaches and techniques (Anastasakis et al., 2007; Ehrig et al., 2009; González Pérez et al., 2012; Macedo and Cunha, 2013), the ability to coverage criteria directed instance generation is still quite limited.

One of the earliest approaches to generating programs using coverage criteria is Purdom's algorithm, based on generating programs that cover all the rules in a context free grammar (CFG) (Purdom, 1972). However, a metamodel captures more than a CFG because the static semantics can be defined, e.g. using extra OCL constraints. Though work has been done on extending Purdom's approach to attribute grammars (Harm and Lämmel, 2000), thus incorporating semantic constraints, the core generation framework is still based on rule coverage, and more general coverage criteria are not considered.

The most closely related research to our work is the model finding tool *Alloy* (Jackson, 2002). Alloy translates a relational specification into formulas for a SAT solver, and each successful assignment for the SAT instances can be mapped back to the problem domain, and much of the research built around Alloy facilitates instance generation. In previous work we have used Alloy to generate instances of metamodels using the Eclipse Modeling Framework, and then applied test-suite reduction techniques in order to pick out instances contributing to a coverage criteria (McQuillan and Power, 2008). However much of our work (and that of others) was limited by the capabilities of Alloy, particularly in relation to coverage oriented generation and quantitative constraints (Anastasakis et al., 2007; Bordbar and Anastasakis, 2005; Kuhlmann et al., 2011; Sen et al., 2009; Kuhlmann and Gogolla, 2012).

The latest version of Alloy employs a powerful relational engine called *kodkod* (Torlak and Jackson, 2007), which outperforms previous versions of Alloy

on large-scale problem solving. However, a major disadvantage is the dependence on SAT solvers which perform poorly when dealing with numeric quantities, using calculations such as addition, multiplication, comparison, etc.

Graph grammars offer a natural way to describe the instance generation process and so have an advantage for generating metamodel instances (Ehrig et al., 2009; Hoffmann and Minas, 2011). Though graph grammars deal with graphs, it is more difficult to use their grammars to quantify a set of nodes than using first-order logic. Parsing a graph is expensive because graph matching is not always deterministic. Thus, the cost of using graph grammars to produce instances that meet graph-based constraint could be very high.

Cabot et al. propose a detailed systematic procedure that reduces the problem of UML class diagram instantiation to a Constraint Satisfaction Problem (CSP) (González Pérez et al., 2012; Cabot et al., 2008; Cabot et al., 2014). The main advantage is that CSP provides a high-level language so that a particular constraint problem is programmable. Our approach distinguishes with theirs by reducing it to an SMT problem. SMT encoding provides a much better expressiveness power than SAT, and it is more natural to encode a problem into SMT formulas. Much research has been made in improving SMT solvers' expressiveness and performance (Barrett et al., 2010; De Moura and Bjørner, 2008; Barrett et al., 2011; Cimatti et al., 2013), which make them more suitable for complicated tasks such as verification, test case generation, program synthesis, etc (Büttner et al., 2012; Felbinger and Schwarzl, 2014; Tillmann and De Halleux, 2008; Gulwani, 2010).

Soeken et al. encode a UML class diagram in a set of operations on bit-vectors which can be solved by SMT solvers using bit-vector theory (Soeken et al., 2010). A successful assignment for each bit-vector can be interpreted as an instance of the UML class diagram. They also propose an approach to encode a subset of OCL constraints as bit-vectors, and provide a list of corresponding mappings between OCL collection data types and bit-vector operations (Soeken et al., 2011). However, their approach does not support structural constraints on the metamodel, especially the quantitative constraints for associations. Furthermore, it is unlikely to use their approach to generate instances satisfying graph-based constraints because they do not represent a metamodel as a graph and provide no tool support.

7 Conclusion

In this paper, we have presented a new approach to improve metamodel instance generation by considering two kinds of coverage criteria: standard coverage criteria defined for UML class diagram and graph-based criteria. Both kinds of criteria are translated to SMT formulas and solved using an external SMT solver. We have already implemented and integrated our techniques into a tool, and results reveal both its capabilities and limitations. In the future, we plan to improve expressiveness of graph properties to allow users to be able to describe more complicated graph shapes. We will also investigate a way of detecting the conflicts between coverage criteria and OCL invariants defined for a metamodel.

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